6p-

X63-11488

NASA TT F-8150 code-2d

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by M. Fenain

FACILITY FORM 602

N71-71253 (ACCESSION NUMBER)

(THRU)

6 (PAGES)

None (CODE)

(NASA CR OR TMX OR AD NUMBER)

(CATEGORY)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

January 1962

NASA TT F-8150

JAN 1 8 1962

DRAFT TRANSLATION

CALCULATION OF WINGS OF A FORM IN ANY PLANE IN SUPERSONIC REGIME

(Calcul des ailes de forme en plan quelconque en regime supersonique)

(FRANCE)

Comptes -Rendus de l'Acad.des Sc.
T.253, NO. 22, pp. 2634-2636,
Paris, Dec. 1961.

by M. FENAIN

(Presented by M. Maurice Roy)

1. In a preceding paper [1]*, we have expounded a method for the solution of a linearized equation governing flows about three-dimensional obstacles in supersonic regime. We shall apply this method here as an example of certain problems of wings only. These wings (or airfoils), near the plane $y = 0$, and symmetrical in respect to the plane $x = 0$, are such that the projection of the leading edge on $y = 0$ has for equation $|x| = h(t)$ with $h(o) = 0$ and $h(1) = 1$, $h(t)$ being of the order of t^m ($m \geqslant 1$) for $t \simeq 0$.

2. For a thick wing, without incidence and symmetric in relation to the plane $y = 0$, we note $lg(t, x)$, the mark of the surface "in extrados". If $|x| \leqslant h(t)$, $w(t, x) = \partial g / \partial t$ and if $|x| > h(t)$, $w = 0$, provided $y = +o$.

We pose $w = w^*(t, x)$ on the wing, or what amounts to the same, $g(t, x)$. This datum, which is marked on the plan $y = o$, is independent from ε. For a given $\underline{t}$, $w^*(t, x)$ is a function of $\underline{x}$ which may presents jumps for $|x| \leqslant h$.

At the first iteration step, the potential is given in the inner

expansion by $F_{o,o}$ such as $\Delta F_{o,o} = 0$. If $F_{o,o}$ is decomposed into two harmonic terms $P_{o,o}$ and $Q_{o,o}$, and if $P_{o,o} = RE\left[\mathcal{F}_{o,o}(z)\right]$ satisfies the data in the plane $y = 0$, we have

$$\mathcal{F}_{o,o} = \frac{1}{\pi}\int_{-h(t)}^{h(t)} w^*(t,\xi)\log(z-\xi)\,d\xi. \qquad (1)$$

The polyharmonic terms are decomposed in the same fashion

$$F_{\nu,o} = P_{\nu,o} + Q_{\nu,o} = \mathrm{Re}\left[\mathcal{F}_{\nu,o}(z,\bar{z}) + \mathcal{Q}_{\nu,o}(z,\bar{z})\right].$$

Let us note $L_\nu(x)$ the ν-th primitive of $\log x$, say

$$L_\nu(x) = \frac{x^\nu}{\nu!}(\log x - s_\nu), \quad \text{with} \quad s_\nu = \sum_{p=1}^{\nu}\frac{1}{p} \qquad (s_o = o).$$

The terms $\mathcal{F}_{\nu,o}$ given by

$$\mathcal{F}_{\nu,o} = \frac{1}{\pi}\frac{\partial^{2\nu}}{\partial t^{2\nu}}\int_{-h}^{h} w^*(t,\xi)\frac{(z-\xi)^\nu}{\nu!}L_\nu(z-\xi)\,d\xi \qquad (\nu \geq o) \qquad (2)$$

verify the intercompatibility equations $\partial^2\mathcal{F}_{\nu,o}/\partial z\,\partial\bar{z} = (\partial^2/\partial t^2)\,\mathcal{F}_{\nu-1,o}$.

They are such that $\overset{w}{P}_{\nu,o} = -\mathrm{Im}\left(\partial\mathcal{F}_{\nu,o}/\partial z - \partial\mathcal{F}_{\nu,o}/\partial\bar{z}\right) = o$ for $y = 0$ if $\nu \geq 1$. Therefore the $P_{\nu,o}$ $(\nu \geq 1)$ do not disturb the data at limits.

From the comparison of expansions for a great $\underline{r}$ of $P_{\nu,o}$ and $F_{\nu,o}$ we deduce that $\beta_{n,\nu,o} = 0$, and that

$$\alpha_{n,\nu,o} = \frac{-\varepsilon_n''}{\pi}\,\frac{1}{\nu!(\nu+n)!}\,\frac{d^{2\nu}}{dt^{2\nu}}\int_{-h}^{h} w^*(t,\xi)\,\xi^{\nu+n}\,d\xi. \qquad (3)$$

where $\varepsilon_o^o = 1$ and $\varepsilon_o^n = 2$ for $n \neq 0$. Since $w^*(t,-\xi) = w^*(t,\xi)$, the $\alpha_{2p+1,\nu,o}$ are zero.

We determine the $Q_{\nu,o}$ by effecting the difference of $F_{\nu,o}$ and $P_{\nu,o}$ expansions, whence

$$Q_{\nu,o} = \sum_{l=0}^{\nu} \sum_{n=0}^{l} \frac{(-1)^n r^{2l-n} \cos n\theta}{l!\,(l-n)!} \frac{d^{2l+1}}{dt^{2l+1}} [\log t \star \alpha_{n,\nu-l,o}(t)]. \qquad (4)$$

We verify that these terms do not modify the data at limits.

We also may express $Q_{\nu,o}$ by iteration starting from $Q_{o,o}$. Since

$$Q_{o,o} = \mathfrak{Q}_{o,o} = -\frac{1}{\pi} \frac{d}{dt} \left[\log t \star \int_{-h}^{h} w^*(t,\xi)\, d\xi \right],$$

we find

$$Q_{\nu,o} = \mathfrak{Q}_{\nu,o} = -\frac{1}{\pi} \frac{\partial^{2\nu-1}}{\partial t^{2\nu-1}} \int_0^t \log(t-\tau)\, d\tau \int_{-h(\tau)}^{h(\tau)} w^*(\tau,\xi) \frac{(\tau-\xi)^\nu}{\nu!} \frac{(\tau-\xi)^\nu}{\nu!} d\xi. \qquad (5)$$

For $\mu \neq 0$, the $F_{\nu,\mu}$ are calculated by the same procedures. Thus, for $\mu = 1$, the iterative method gives, starting from $F_{o,1} = -\alpha_{o,o,o}$

$$F_{\nu,1} = \frac{1}{\pi} \frac{\partial^{2\nu}}{\partial t^{2\nu}} \int_{-h}^{h} w^*(t,\xi) \frac{(\tau-\xi)^\nu}{\nu!} \frac{(\tau-\xi)^\nu}{\nu!} d\xi. \qquad (6)$$

By identifying with the development of $F_{\nu,1}$ for $\underline{r}$ great, we find [*]

The identification also renders the $\gamma_{n,\nu,1}$ more precise. We necessarily have $\gamma_{n,\nu,1} = 0$. The data at limits are not perturbed, and the $F_{\nu,1}$ include only bounded (restricted) terms.

Taking into account the different conditions to be satisfied, we progressively show, that $F_{\nu,\mu} = 0$ for $\mu \geq 2$.

In the final resort, for a thick wind, doubly symmetric and without incidence, the form of which in the plane is limited by $|x| = h(t)$, and whose slope is in the direction of the flow is given by $w^*(t,x)$, the

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[*] inserting formula (7) $F_{\nu,1} = -\sum_{l=0}^{\nu} \sum_{n=0}^{l} \frac{(-1)^n r^{2l-n} \cos n\theta}{l!\,(l-n)!} \alpha^{(2l)}_{n,\nu-l,o}$. (7)

perturbation potential is completely known in an expanded form by

$$\varphi = bF = b \sum_{\nu=0} \left(\frac{\xi}{2}\right)^{2\nu} \left[F_{\nu,0} + \log\left(\frac{\xi}{2}\right) F_{\nu,1} \right], \tag{8}$$

where the $F_{\nu,0}$ and $F_{\nu,1}$ are all expressed in advance, as a function of $h(t)$ and $w^*(t, x)$ (2), (5), (6). Hence, we conclude by derivation that the pressure coefficient is $Cp = -2u = -(2b/l)(\partial F/\partial t)$.

Let us note that $F_{\nu,0}$ and $F_{\nu,1}$, just as the $\alpha_{n,\nu,0}$ may be expressed with the help of $g(t, x)$. Since $g[t, h(t)] = 0$, we have, f.ex.

$$\alpha_{n,\nu,0} = \frac{-\xi_0''}{\pi} \frac{1}{\nu!(\nu+n)!} \frac{d^{2\nu+1}}{dt^{2\nu+1}} \int_{-h}^{h} g(t, \xi) \xi^{2\nu+n} d\xi. \tag{9}$$

If $S = bl\sigma(t)$ is the surface of a transverse section, we may write, in particular, $\alpha_{0,0,0} = -(2\pi)^{-1} \sigma'(t)$.

3. In the case of a wing without thickness, we postulate a potential on the plane $y = 0$. With $F = F^*(t, x)$ for $|x| < h(t)$ and $F = 0$ for $|x| \geq h(t)$, the problem is strictly identical to that which would have been treated in 2) if one were searching for it

$$\overset{w}{F} = w = \frac{\partial F}{\partial y} - i\left(\frac{\partial F}{\partial z} - \frac{\partial F}{\partial \xi}\right).$$

The potential thus is of the form (8) where the $F_{\nu,0}$ and $F_{\nu,1}$ are obtained by derivating the previous ones in relation to y and by substituting F^* for w^*. In this case $\alpha_{n,\nu,0} = 0$ and

$$\beta_{n,\nu,0} = \frac{2}{\pi} \frac{n}{\nu!(\nu+n)!} \frac{d^{2\nu}}{dt^{2\nu}} \int_{-h}^{h} F^*(t, \xi) \xi^{2\nu+n-1} d\xi \qquad (n \geq 1), \tag{10}$$

whence, taking account of the parity of F^*, $\beta_{2p,\nu,0} = 0$.

Deriving F in respect to y we obtain $\underline{w}$ which may be expressed in particular in the plane $y = 0$. The integrations in respect to $\underline{t}$ furnish the mark at each point of the wing.

4. At the application stage, the computations are simplified by the fact that attention is given only to expressions of $\underline{u}$, $\underline{w}$ or $\underline{g}$ in the plane $y = 0$ ($z = \bar{z} = x$).

Certain data, similar to those encountered for the homogenous flows [2], are particularly interesting in the sense, that they allow the determination in advance, of the necessary elements for the calculation of very large families of wings. Such is for instance the case of polynomial data, or of those of the type , of which combinations can be made, whereby arbitraries are still available that lend themselves, for example, to the search of optimum configurations.

*** E N D ***

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2. M. FENAIN, Progress in Aeronautical Sciences, 1, Pergamon Press, 1960

* Nasa Translation

Translated by ANDRE L. BRICHANT
Contractor for NASA HEADQUARTERS
17 January, 1962